Einige wichtige Integrale

Bei Verwendung der Ergebnisse als Stammfunktion ist stets noch die Integrationskonstante hinzuzufügen!

1 Rationale Funktionen:

1.1 $$\int x^n dx = \frac{1}{n+1} \cdot x^{n+1} \qquad (n \neq -1)$$

1.2 $$\int (ax+b)^n dx = \frac{1}{a(n+1)} \cdot (ax+b)^{n+1} \qquad (n \neq -1)$$

1.3 $$\int x \cdot (ax+b)^n dx = \frac{1}{(n+2)a^2}(ax+b)^{n+2} - \frac{b}{(n+1)a^2}(ax+b)^{n+1}$$

1.4 $$\int \frac{1}{x} dx = \ln|x|$$

1.5 $$\int \frac{1}{ax+b} dx = \frac{1}{a} \cdot \ln|ax+b|$$

1.6 $$\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b|$$

1.7 $$\int \frac{1}{x \cdot (ax+b)} dx = -\frac{1}{b} \ln\left|\frac{ax+b}{x}\right|$$

1.8 $$\int \frac{ax+b}{cx+d} dx = \frac{ax}{c} + \frac{bc-ad}{c^2} \ln|cx+d|$$

1.9 $$\int \frac{1}{(ax+b)(cx+d)} dx = \frac{1}{bc-ad} \ln\left|\frac{cx+d}{ax+b}\right| \qquad (bc \neq ad)$$

1.10 $$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \cdot \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} \qquad \text{bei } 4ac > b^2$$

$$= -\frac{2}{2ax+b} \qquad \text{bei } 4ac = b^2$$

$$= \frac{1}{\sqrt{b^2-4ac}} \cdot \ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| = \frac{2}{\sqrt{b^2-4ac}} \cdot \operatorname{arctanh} \frac{2ax+b}{\sqrt{b^2-4ac}} \qquad \text{bei } 4ac < b^2$$

Im Fall $4ac < b^2$ auch (als Spezialfall von 1.9):

$$\int \frac{1}{(x-p)(x-q)} dx = \frac{1}{p-q} \cdot \ln\left|\frac{x-p}{x-q}\right| \qquad (p \neq q)$$

1.11 $$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln\left|ax^2+bx+c\right| - \frac{b}{2a} \cdot \int \frac{1}{ax^2+bx+c} dx \qquad \text{(siehe 1.10)}$$

1.12 $$\int \frac{1}{x \cdot (ax^2+bx+c)} dx = \frac{1}{2c} \ln \frac{x^2}{ax^2+bx+c} - \frac{b}{2c} \cdot \int \frac{1}{ax^2+bx+c} dx \qquad \text{(siehe 1.10)}$$

1.13 $$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a}$$

1.14 $$\int \frac{1}{a^2-x^2} dx = \frac{1}{2} \ln\left|\frac{a+x}{a-x}\right|$$

1.15 $$\int \frac{x}{a^2 \pm x^2} dx = \pm\frac{1}{2} \ln\left|a^2 \pm x^2\right|$$

1.16 $$\int \frac{1}{x \cdot (a^2 \pm x^2)} dx = \frac{1}{2a^2} \ln\left|\frac{x^2}{a^2 \pm x^2}\right|$$

1.17 $$\int \frac{1}{x^2+1} dx = \arctan x$$

1.18 $$\int \frac{1}{x^2-1} dx = \frac{1}{2} \cdot \ln\left|\frac{x-1}{x+1}\right|$$

2 Wurzelfunktionen:

2.1 $$\int \sqrt{x}\, dx = \frac{2}{3} \cdot \sqrt{x^3} = \frac{2}{3} x \sqrt{x}$$

2.2 $$\int x\sqrt{x}\, dx = \frac{2}{5} \sqrt{x^5} = \frac{2}{5} x^2 \sqrt{x}$$

2.3 $$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$$

2.4 $$\int \frac{1}{x\sqrt{x}} dx = -\frac{2}{\sqrt{x}}$$

2.5 $$\int \sqrt{ax+b}\, dx = \frac{2}{3a} \cdot \sqrt{(ax+b)^3}$$

2.6 $\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\cdot\sqrt{(ax+b)^3}$

2.7 $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\cdot\int \frac{1}{x\sqrt{ax+b}}dx$ (siehe 2.10)

2.8 $\int \frac{1}{\sqrt{ax+b}}dx = \frac{2}{a}\cdot\sqrt{ax+b}$

2.9 $\int \frac{x}{\sqrt{ax+b}}dx = \frac{2(ax-2b)}{3a^2}\cdot\sqrt{ax+b}$

2.10 $\int \frac{1}{x\sqrt{ax+b}}dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right|$ bei $b>0$

$= \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}}$ bei $b<0$

2.11 $\int \frac{1}{x^n\sqrt{ax+b}}dx = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\cdot\int \frac{1}{x^{n-1}\sqrt{ax+b}}dx$ (führt letztlich auf 2.10)

2.12 $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a}$

2.13 $\int \sqrt{x^2\pm a^2}\,dx = \frac{x}{2}\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\ln\left|x+\sqrt{x^2\pm a^2}\right|$

2.14 $\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3}$

2.15 $\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2\pm a^2)^3}$

2.16 $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\cdot\arccos\frac{a}{x}$

2.17 $\int \frac{\sqrt{a^2\pm x^2}}{x}dx = \sqrt{a^2\pm x^2} - a\cdot\ln\left|\frac{a+\sqrt{a^2\pm x^2}}{x}\right|$

2.18 $\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a}$

2.19 $\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|$

2.20 $\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}$

2.21 $\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2}$

2.22 $\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\arccos\frac{a}{x}$

2.23 $\int \frac{1}{x\sqrt{a^2\pm x^2}}dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2\pm x^2}}{x}\right|$

2.24 $\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a}\cdot\int \frac{1}{\sqrt{ax^2+bx+c}}dx$ (siehe 2.26)

2.25 $\int x\sqrt{ax^2+bx+c}\,dx = \frac{1}{3a}\sqrt{(ax^2+bx+c)^3} - \frac{2abx+b^2}{8a^2}\sqrt{ax^2+bx+c} - \frac{4abc-b^3}{16a^2}\cdot\int \frac{1}{\sqrt{ax^2+bx+c}}dx$

2.26 $\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2\sqrt{a(ax^2+bx+c)} + 2ax+b\right|$ bei $a>0$

$= \frac{1}{\sqrt{a}}\ln|2ax+b|$ bei $a>0 \wedge 4ac=b^2$

$= -\frac{1}{\sqrt{-a}}\arcsin\frac{2ax+b}{\sqrt{b^2-4ac}}$ bei $a<0 \wedge 4ac<b^2$

2.27 $\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a}\cdot\int \frac{1}{\sqrt{ax^2+bx+c}}dx$ (siehe 2.26)

3 Sinus-Funktionen

3.1 $\int \sin x\,dx = -\cos x$

3.2 $\int \sin ax\,dx = -\frac{1}{a}\cos ax$

3.3 $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax$

3.4 $\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax \cdot \cos ax}{n\,a} + \frac{n-1}{n} \cdot \int \sin^{n-2} ax\,dx$ ($n \geq 2$, ganz; führt letztlich auf 3.2)

3.5 $\int x \cdot \sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x \cdot \cos ax}{a}$

3.6 $\int x^n \cdot \sin ax\,dx = -\frac{x^n}{a}\cos ax + \frac{n}{a} \cdot \int x^{n-1} \cdot \cos ax\,dx$ ($n \geq 1$, ganz; siehe 4.6)

3.7 $\int \frac{1}{\sin ax}dx = \frac{1}{a}\ln\left|\tan\frac{ax}{2}\right|$

3.8 $\int \frac{1}{\sin^2 ax}dx = -\frac{1}{a}\cot ax$

3.9 $\int \frac{1}{\sin^n ax}dx = -\frac{1}{a(n-1)}\frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \cdot \int \frac{1}{\sin^{n-2} ax}dx$ ($n \geq 2$, ganz; führt letztlich auf 3.7)

3.10 $\int \frac{1}{1 \pm \sin ax}dx = \frac{1}{a}\tan\left(\frac{ax}{2} \mp \frac{\pi}{4}\right)$

3.11 $\int \frac{\sin ax}{1 \pm \sin ax}dx = \pm x + \frac{1}{a}\tan\left(\frac{\pi}{4} \mp \frac{ax}{2}\right)$

3.12 $\int \frac{x}{1 + \sin ax}dx = -\frac{x}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\cos\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right|$

3.13 $\int \frac{x}{1 - \sin ax}dx = \frac{x}{a}\cot\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\sin\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right|$

3.14 $\int \sin ax \cdot \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}$ ($|a| \neq |b|$)

4 Cosinus-Funktionen

4.1 $\int \cos x\,dx = \sin x$

4.2 $\int \cos ax\,dx = \frac{1}{a}\sin ax$

4.3 $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax$

4.4 $\int \cos^n ax\,dx = \frac{\cos^{n-1} ax \cdot \sin ax}{n\,a} + \frac{n-1}{n} \cdot \int \cos^{n-2} ax\,dx$ ($n \geq 2$, ganz; führt letztlich auf 4.2)

4.5 $\int x \cdot \cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x \cdot \sin ax}{a}$

4.6 $\int x^n \cdot \cos ax\,dx = \frac{x^n}{a}\sin ax - \frac{n}{a} \cdot \int x^{n-1} \cdot \sin ax\,dx$ ($n \geq 1$, ganz; siehe 3.6)

4.7 $\int \frac{1}{\cos ax}dx = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right|$

4.8 $\int \frac{1}{\cos^2 ax}dx = \frac{1}{a}\tan ax$

4.9 $\int \frac{1}{\cos^n ax}dx = \frac{1}{a(n-1)}\frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \cdot \int \frac{1}{\cos^{n-2} ax}dx$ ($n \geq 2$, ganz; führt letztlich auf 4.7)

4.10 $\int \frac{1}{1 + \cos ax}dx = \frac{1}{a}\tan\frac{ax}{2}$ $\int \frac{1}{1 - \cos ax}dx = -\frac{1}{a}\cot\frac{ax}{2}$

4.11 $\int \frac{\cos ax}{1 + \cos ax}dx = x - \frac{1}{a}\tan\frac{ax}{2}$ $\int \frac{\cos ax}{1 - \cos ax}dx = -x - \frac{1}{a}\cot\frac{ax}{2}$

4.12 $\int \frac{x}{1 + \cos ax}dx = \frac{x}{a}\tan\frac{ax}{2} + \frac{2}{a^2}\ln\left|\cos\frac{ax}{2}\right|$

4.13 $\int \frac{x}{1 - \cos ax}dx = -\frac{x}{a}\cot\frac{ax}{2} + \frac{2}{a^2}\ln\left|\sin\frac{ax}{2}\right|$

4.14 $\int \cos ax \cdot \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}$ ($|a| \neq |b|$)

5 Sinus- und Cosinusfunktionen gemischt

5.1 $\int \sin ax \cdot \cos ax\, dx = \frac{1}{2a} \sin^2 ax$

5.2 $\int \sin^2 ax \cdot \cos^2 ax\, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$

5.3 $\int \sin^n ax \cdot \cos ax\, dx = \frac{1}{a(n+1)} \sin^{n+1} ax$ $(n \neq -1)$

5.4 $\int \sin ax \cdot \cos^n ax\, dx = -\frac{1}{a(n+1)} \cos^{n+1} ax$ $(n \neq -1)$

5.5 $\int \frac{\sin^n ax}{\cos ax} dx = -\frac{\sin^{n-1} ax}{a(n-1)} + \int \frac{\sin^{n-2} ax}{\cos ax} dx$ $(n \neq 1$; führt letztlich zu 4.7 oder 6.2)

5.6 $\int \frac{\cos^n ax}{\sin ax} dx = \frac{\cos^{n-1} ax}{a(n-1)} + \int \frac{\cos^{n-2} ax}{\sin ax} dx$ $(n \neq 1$; führt letztlich zu 3.7 oder 7.2)

5.7 $\int \frac{1}{\sin ax \cdot \cos ax} dx = \frac{1}{a} \ln|\tan ax|$

5.8 $\int \frac{1}{\sin ax \pm \cos ax} dx = \frac{1}{a\sqrt{2}} \ln\left|\tan\left(\frac{ax}{2} \pm \frac{\pi}{8}\right)\right|$

5.9 $\int \frac{\sin ax}{\sin ax \pm \cos ax} dx = \frac{x}{2} \mp \frac{1}{2a} \ln|\sin ax \pm \cos ax|$

5.10 $\int \frac{\cos ax}{\sin ax \pm \cos ax} dx = \pm\frac{x}{2} + \frac{1}{2a} \ln|\sin ax \pm \cos ax|$

5.11 $\int \frac{\sin ax}{b \cdot \cos ax + c} dx = -\frac{1}{ab} \ln|b \cdot \cos ax + c|$

5.12 $\int \frac{\cos ax}{b \cdot \sin ax + c} dx = \frac{1}{ab} \ln|b \cdot \sin ax + c|$

5.13 $\int \sin ax \cdot \cos bx\, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$ $(|a| \neq |b|)$

6 Tangensfunktionen

6.1 $\int \tan x\, dx = -\ln|\cos x|$

6.2 $\int \tan ax\, dx = -\frac{1}{a} \ln|\cos ax|$

6.3 $\int \tan^2 ax\, dx = \frac{\tan ax}{a} - x$

6.4 $\int \tan^n ax\, dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax\, dx$ (führt letztlich zu 6.2 oder 6.3)

6.5 $\int \frac{1}{\tan ax \pm 1} dx = \pm\frac{x}{2} + \frac{1}{2a} \ln|\sin ax \pm \cos ax|$

6.6 $\int \frac{\tan ax}{\tan ax \pm 1} dx = \frac{x}{2} \mp \frac{1}{2a} \ln|\sin ax \pm \cos ax|$

7 Cotangensfunktionen

7.1 $\int \cot x\, dx = \ln|\sin x|$

7.2 $\int \cot ax\, dx = \frac{1}{a} \ln|\sin ax|$

7.3 $\int \cot^2 ax\, dx = -\frac{\cot ax}{a} - x$

7.4 $\int \cot^n ax\, dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax\, dx$ (führt letztlich zu 7.2 oder 7.3)

7.5 $\int \frac{1}{1 \pm \cot ax} dx = \int \frac{\tan ax}{\tan ax \pm 1} dx$ (siehe 6.6)

8 Exponentialfunktionen

8.1 $\int e^x dx = e^x$

8.2 $\int e^{ax} dx = \frac{1}{a} e^{ax}$

8.3 $\int x \cdot e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$

8.4 $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \cdot \int x^{n-1} e^{ax} dx$ (führt letztlich zu 8.3)

8.5 $\int \frac{1}{b \cdot e^{ax} + c} dx = \frac{x}{c} - \frac{1}{ac} \ln|b \cdot e^{ax} + c|$

8.6 $\int \frac{e^{ax}}{b \cdot e^{ax} + c} dx = \frac{1}{ab} \ln|b \cdot e^{ax} + c|$

9 Logarithmusfunktionen

9.1 $\int \ln x \, dx = x \cdot \ln x - x$

9.2 $\int (\ln x)^2 dx = x \cdot (\ln x)^2 - 2x \cdot \ln x + 2x$

9.3 $\int (\ln x)^n dx = x \cdot (\ln x)^n - n \cdot \int (\ln x)^{n-1} dx$ ($n \neq -1$; führt letztlich zu 9.1)

9.4 $\int x^m \ln x \, dx = x^{m+1} \left[\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right]$ ($m \neq -1$)

9.5 $\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1}$

9.6 $\int \frac{\ln x}{x^m} dx = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2 x^{m-1}}$ ($m \neq 1$)

9.7 $\int \frac{(\ln x)^n}{x^m} dx = -\frac{(\ln x)^n}{(m-1)x^{m-1}} + \frac{n}{m-1} \cdot \int \frac{(\ln x)^{n-1}}{x^m} dx$ ($m \neq 1$, führt letztlich zu 9.6)

9.8 $\int \frac{1}{x \cdot \ln x} dx = \ln \ln x$

9.9 $\int \frac{1}{x \cdot (\ln x)^n} dx = -\frac{1}{(n-1)(\ln x)^{n-1}}$ ($n \neq 1$)

10 Inverse trigonometrische Funktionen

10.1 $\int \arcsin x \, dx = x \cdot \arcsin x + \sqrt{1-x^2}$ $\int \arccos x \, dx = x \cdot \arccos x - \sqrt{1-x^2}$

$\int \arctan x \, dx = x \cdot \arctan x - \frac{1}{2} \ln(1+x^2)$ $\int \operatorname{arccot} x \, dx = x \cdot \operatorname{arccot} x + \frac{1}{2} \ln(1+x^2)$

10.2 $\int \arcsin \frac{x}{a} \, dx = x \cdot \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}$

10.3 $\int x \cdot \arcsin \frac{x}{a} \, dx = \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \arcsin \frac{x}{a} + \frac{x}{4} \sqrt{a^2 - x^2}$

10.4 $\int \arccos \frac{x}{a} \, dx = x \cdot \arccos \frac{x}{a} - \sqrt{a^2 - x^2}$

10.5 $\int x \cdot \arccos \frac{x}{a} \, dx = \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \arccos \frac{x}{a} - \frac{x}{4} \sqrt{a^2 - x^2}$

10.6 $\int \arctan \frac{x}{a} \, dx = x \cdot \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2)$

10.7 $\int x \cdot \arctan \frac{x}{a} \, dx = \frac{1}{2}(x^2 + a^2) \arctan \frac{x}{a} - \frac{ax}{2}$

10.8 $\int \operatorname{arccot} \frac{x}{a} \, dx = x \cdot \operatorname{arccot} \frac{x}{a} + \frac{a}{2} \ln(a^2 + x^2)$

10.9 $\int x \cdot \operatorname{arccot} \frac{x}{a} \, dx = \frac{1}{2}(x^2 + a^2) \operatorname{arccot} \frac{x}{a} + \frac{ax}{2}$

11 Hyperbelfunktionen

11.1 $\int \sinh x\,dx = \cosh x$ $\qquad$ $\int \cosh x\,dx = \sinh x$

$\int \tanh x\,dx = \ln \cosh x$ $\qquad$ $\int \coth x\,dx = \ln \sinh x$

11.2 $\int \sinh ax\,dx = \frac{1}{a}\cosh ax$

11.3 $\int \sinh^2 ax\,dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2}$

11.4 $\int \frac{1}{\sinh ax}dx = \frac{1}{a}\ln\left|\tanh\frac{ax}{2}\right|$

11.5 $\int \frac{1}{\sinh^2 ax}dx = -\frac{1}{a}\coth ax$

11.6 $\int x\cdot\sinh ax\,dx = \frac{x}{a}\cosh ax - \frac{1}{a^2}\sinh ax$

11.7 $\int \sinh ax\cdot\sinh bx\,dx = \frac{1}{a^2-b^2}(a\cdot\sinh bx\cdot\cosh ax - b\cdot\sinh ax\cdot\cosh bx)$ $\qquad (|a|\neq|b|)$

11.8 $\int \cosh ax\,dx = \frac{1}{a}\sinh ax$

11.9 $\int \cosh^2 ax\,dx = \frac{1}{4a}\sinh 2ax + \frac{x}{2}$

11.10 $\int \frac{1}{\cosh ax}dx = \frac{2}{a}\arctan e^{ax}$

11.11 $\int \frac{1}{\cosh^2 ax}dx = \frac{1}{a}\tanh ax$

11.12 $\int x\cdot\cosh ax\,dx = \frac{x}{a}\sinh ax - \frac{1}{a^2}\cosh ax$

11.13 $\int \cosh ax\cdot\cosh bx\,dx = \frac{1}{a^2-b^2}(a\cdot\sinh ax\cdot\cosh bx - b\cdot\sinh bx\cdot\cosh ax)$ $\qquad (|a|\neq|b|)$

11.14 $\int \tanh ax\,dx = \frac{1}{a}\ln\cosh ax$

11.15 $\int \tanh^2 ax\,dx = x - \frac{\tanh ax}{a}$

11.16 $\int \coth ax\,dx = \frac{1}{a}\ln\sinh ax$

11.17 $\int \coth^2 ax\,dx = x - \frac{\coth ax}{a}$

11.18 $\int \sin ax\cdot\cosh bx\,dx = \frac{1}{a^2-b^2}(b\cdot\sinh ax\cdot\sinh bx - a\cdot\cos ax\cdot\cosh bx)$ $\qquad (|a|\neq|b|)$

12 Inverse Hyperbelfunktionen

12.1 $\int \operatorname{arsinh} x\,dx = x\cdot\operatorname{arsinh} x - \sqrt{x^2+1}$ $\qquad$ $\int \operatorname{arcosh} x\,dx = x\cdot\operatorname{arcosh} x - \sqrt{x^2-1}$

$\int \operatorname{artanh} x\,dx = x\cdot\operatorname{artanh} x + \frac{1}{2}\ln(1-x^2)$ $\qquad$ $\int \operatorname{arcoth} x\,dx = x\cdot\operatorname{arcoth} x + \frac{1}{2}\ln(x^2-1)$

12.2 $\int \operatorname{arsinh}\frac{x}{a}\,dx = x\cdot\operatorname{arsinh}\frac{x}{a} - \sqrt{x^2+a^2}$

12.3 $\int \operatorname{arcosh}\frac{x}{a}\,dx = x\cdot\operatorname{arcosh}\frac{x}{a} - \sqrt{x^2-a^2}$

12.4 $\int \operatorname{artanh}\frac{x}{a}\,dx = x\cdot\operatorname{artanh}\frac{x}{a} + \frac{a}{2}\ln(a^2-x^2)$

12.5 $\int \operatorname{arcoth}\frac{x}{a}\,dx = x\cdot\operatorname{arcoth}\frac{x}{a} + \frac{a}{2}\ln(x^2-a^2)$

13 Gemischte Funktionen

13.1 $\int e^{ax} \sin\ bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \cdot \sin\ bx - b \cdot \cos\ bx)$

13.2 $\int e^{ax} \cos\ bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \cdot \cos\ bx + b \cdot \sin\ bx)$

13.3 $\int e^{ax} \sin^n x\, dx = \frac{e^{ax} \sin^{n-1} x}{a^2+n^2}(a \cdot \sin\ x - n \cdot \cos\ x) + \frac{n(n-1)}{a^2+n^2} \cdot \int e^{ax} \sin^{n-2} x\, dx$ (s. 8.2 oder 13.1)

13.4 $\int e^{ax} \cos^n x\, dx = \frac{e^{ax} \cos^{n-1} x}{a^2+n^2}(a \cdot \cos\ x + n \cdot \sin\ x) + \frac{n(n-1)}{a^2+n^2} \cdot \int e^{ax} \cos^{n-2} x\, dx$ (s. 8.2 oder13.2)

13.5 $\int x \cdot e^{ax} \sin\ bx\, dx = \frac{x \cdot e^{ax}}{a^2+b^2}(a \cdot \sin\ bx - b \cdot \cos\ bx) - \frac{e^{ax}}{(a^2+b^2)^2}\left[(a^2-b^2)\sin\ bx - 2ab \cdot \cos\ bx\right]$

13.6 $\int x \cdot e^{ax} \cos\ bx\, dx = \frac{x \cdot e^{ax}}{a^2+b^2}(a \cdot \cos\ bx + b \cdot \sin\ bx) - \frac{e^{ax}}{(a^2+b^2)^2}\left[(a^2-b^2)\cos\ bx - 2ab \cdot \sin\ bx\right]$

13.7 $\int \sin\ \ln\ x\, dx = \frac{x}{2}(\sin\ \ln\ x - \cos\ \ln\ x)$

13.8 $\int \cos\ \ln\ x\, dx = \frac{x}{2}(\sin\ \ln\ x + \cos\ \ln\ x)$

13.9 $\int \sinh\ ax \cdot \sin\ ax\, dx = \frac{1}{2a}(\cosh\ ax \cdot \sin\ ax - \sinh\ ax \cdot \cos\ ax)$

13.10 $\int \cosh\ ax \cdot \cos\ ax\, dx = \frac{1}{2a}(\sinh\ ax \cdot \cos\ ax + \cosh\ ax \cdot \sin\ ax)$

13.11 $\int \sinh\ ax \cdot \cos\ ax\, dx = \frac{1}{2a}(\cosh\ ax \cdot \cos\ ax + \sinh\ ax \cdot \sin\ ax)$

13.12 $\int \cosh\ ax \cdot \sin\ ax\, dx = \frac{1}{2a}(\sinh\ ax \cdot \sin\ ax - \cosh\ ax \cdot \cos\ ax)$
